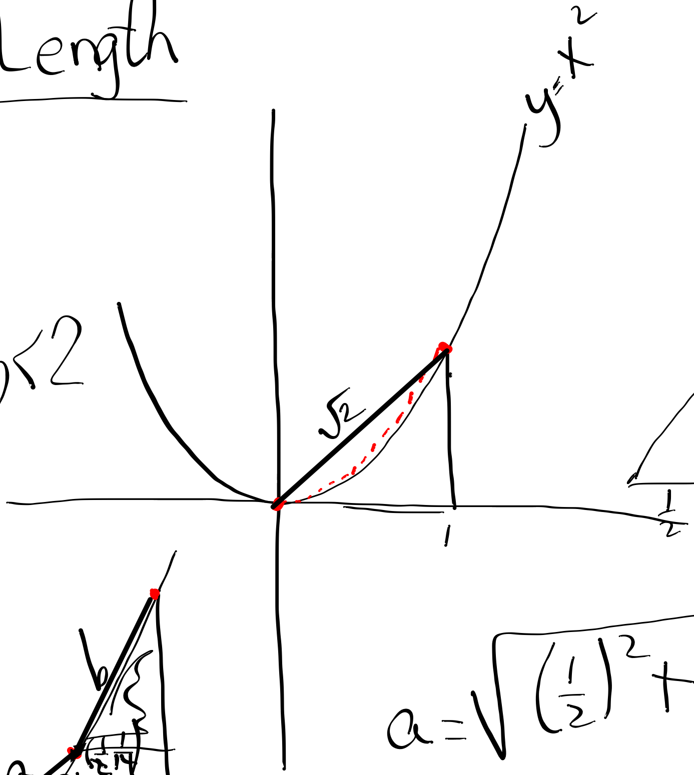
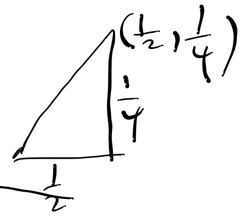
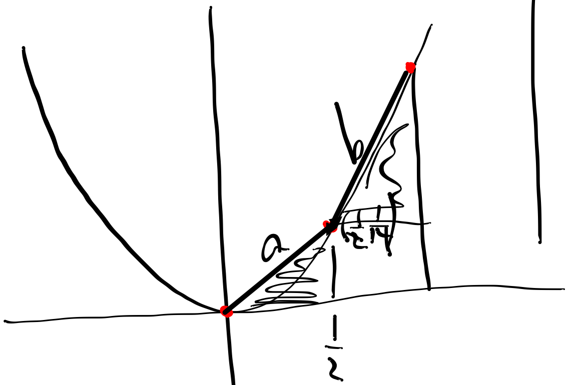
7.4) Arc Length





$\sqrt{2} < \text{arclength} < 2$



$$a = \sqrt{\left(\tfrac{1}{2}\right)^2 + \left(f\left(\tfrac{1}{2}\right)\right)^2}$$

$$= \sqrt{\tfrac{1}{4} + \tfrac{1}{16}} = \frac{\sqrt{5}}{4} \approx .559$$

$$b = \sqrt{\left(\tfrac{1}{2}\right)^2 + \left(f(1) - f\left(\tfrac{1}{2}\right)\right)^2}$$

$$= \sqrt{\tfrac{1}{4} + \left(\tfrac{3}{4}\right)^2}$$

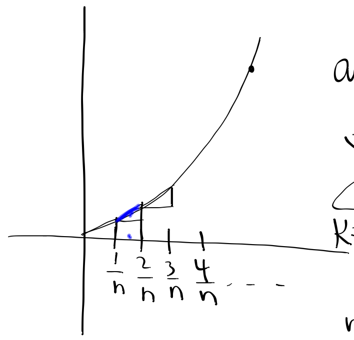
$$= \sqrt{\tfrac{13}{16}}$$

$1.461 < \text{arclength} < 2$

$$\begin{array}{r} a+b \approx .902 \\ + .559 \\ \hline 1.461 \end{array}$$

$$a^2 + b^2 = c^2$$

$$\therefore c = \sqrt{a^2 + b^2}$$



arclength $\approx$

$$\sum_{k=1}^{n} \sqrt{(\Delta x_k)^2+\underbrace{(f(x_k)-f(x_{k-1}))^2}_{(\Delta y_k)^2}}$$

$$\text{arclength} = \lim_{n\to\infty} \sum_{k=1}^{n} \sqrt{\Delta x_k^2+(f(x_k)-f(x_{k-1}))^2}$$

Assume that $f(x)$ is continuous & differentiable on $[a,b]$

then [[by the mean value theorem]]

$$\left(\frac{f(x_k)-f(x_{k-1})}{\Delta x_k}\right)=\left(f'(x_k^*)\right)$$

therefore... $\underbrace{f(x_k)-f(x_{k-1})}_{\text{"}\Delta y_k\text{"}} = f'(x_k^*)\Delta x_k$

$$\text{Arc length} = \lim_{n\to\infty} \sum_{k=1}^{n} \sqrt{(\Delta x_k)^2+(f'(x_k^*)\Delta x_k)^2}$$

$$= \lim_{n\to\infty} \sum_{k=1}^{n} \sqrt{\Delta x_k^2}\sqrt{1+(f'(x_k^*))^2}$$

$$= \lim_{n\to\infty} \sum_{k=1}^{n} \underbrace{\sqrt{1+(f'(x_k^*))^2}}_{\text{thing 1}}\ \underbrace{\Delta x_k}_{\text{thing 2}}$$

$$= \int_a^b \sqrt{1+[f'(x)]^2}\,dx$$

Δx_k $f(x_k^*)$

width $\to 0$ height $\to \neq 0$

$$\text{Arc Length} = \int_a^b \sqrt{1+[f'(x)]^2}\,dx$$

use this to figure out:

1) Arclength of $y=3$ from $x=1$ to $x=4$

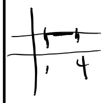
2) Arclength of $y=3x+5$ from $x=1$ to $x=4$

3) Arclength of $y=x^2$ from $x=1$ to $x=2$ [use calculator]

$$\text{Arc Length} = \int_a^b \sqrt{1+[f'(x)]^2}\,dx$$

use this to figure out:

1) Arclength of $y=3$ from $x=1$ to $x=4$



$y=3$

$y'=0$

$$\text{Arclength} = \int_1^4 \sqrt{1+0^2}\,dx = \int_1^4 1\,dx$$

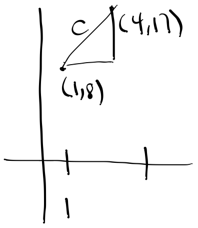
$$= x\Big|_1^4 = 4-1 = 3$$

2) Arclength of $y=3x+5$ from $x=1$ to $x=4$

$y=3x+5$

$y'=3$

$$AL = \int_1^4 \sqrt{1+3^2}\,dx = \int_1^4 \sqrt{10}\,dx$$



$$= \sqrt{10}\,x\Big|_1^4 = 4\sqrt{10} - 1\sqrt{10} = 3\sqrt{10}$$

$c = \sqrt{3^2+9^2} = \sqrt{90}$

$= \sqrt{9}\sqrt{10} = 3\sqrt{10}$

3) Arclength of $y=x^2$ from $x=1$ to $x=2$ [use calculator]

$y=x^2$

$y'=2x$

$$AL = \int_1^2 \sqrt{1+(2x)^2}\,dx$$

$$= \int_1^2 \sqrt{1+4x^2}\,dx$$

[[$\text{fnInt}(\sqrt{1+4x^2}, x, 1, 2) \approx 3.168\ldots$]]

Feb 15-7:35 AM

Arclength $y=x^2$ from 0 to 1

$$A_L = \int_0^1 \sqrt{1+4x^2}\,dx \approx 1.479$$

HW// 7.4/3-8

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8.2/35) $\star \int_2^4 \sec^{-1}\sqrt{\theta}\, d\theta$

$\frac{d}{d\theta}\sqrt{\theta} = \frac{d}{d\theta}\theta^{1/2} = \frac{1}{2}\theta^{-1/2}$

$u = \sec^{-1}\sqrt{\theta}$ $\quad dv = d\theta$

$du = \frac{1}{|\sqrt{\theta}|\sqrt{(\sqrt{\theta})^2 - 1}}\left(\frac{1}{2\sqrt{\theta}}\right)d\theta$ $\quad v = \theta$

$\star = \theta \sec^{-1}\sqrt{\theta}\Big|_2^4 - \int_2^4 \theta\left(\frac{1}{\theta \cdot 2\sqrt{\theta - 1}}\right) d\theta$

$= \underbrace{\theta \sec^{-1}\sqrt{\theta}\Big|_2^4}_{} - \frac{1}{2}\int_2^4 \frac{1}{\sqrt{\theta - 1}}\, d\theta$

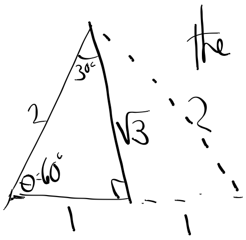
$u = \theta - 1$ $\Rightarrow \int \frac{1}{\sqrt{u}}\, du$

$du = \theta$

$4\sec^{-1}2 - 2\sec^{-1}\sqrt{2} - \ldots$

$\sec^{-1}2$ = the angle whose secant is 2

i.e.

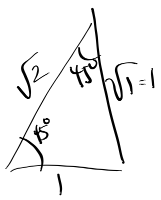


the angle whose cosine is $\frac{1}{2}$

i.e.

$60° = \frac{\pi}{3}$

$\sec^{-1}(\sqrt{2})$ = the angle whose secant is $\sqrt{2}$



= the angle whose cosine is $\frac{1}{\sqrt{2}}$

= the $\measuredangle$ whose cosine is $\frac{\sqrt{2}}{2}$

$= 45° = \frac{\pi}{4}$

$\sec^{-1}(x) = \cos^{-1}\left(\frac{1}{x}\right)$

8.2/40 $\star = \int_0^2 \ln(x^2+1)\,dx$

$u = \ln(x^2+1) \qquad dv = dx$

$du = \frac{1}{x^2+1}(2x) \qquad v = x$

$\star = x\ln(x^2+1)\Big|_0^2 - \int_0^2 \frac{2x^2}{x^2+1}\,dx$

$$\begin{array}{r} 2 \\ x^2+1 \overline{)\,2x^2} \\ -(2x^2+2) \\ \hline -2 \end{array}$$

$= 2\ln 5 - 0 - \int_0^2 2 - 2\left(\frac{1}{x^2+1}\right) dx$

$= 2\ln 5 - \int_0^2 2\,dx + 2\int_0^2 \frac{1}{x^2+1}\,dx$

$= 2\ln 5 - 2x\Big|_0^2 + 2\tan^{-1}x\Big|_0^2$

7.4/4) $x = \frac{1}{3}(y^2+2)^{3/2}$ $y=0$ to $y=1$

$$\frac{dx}{dy} = \frac{1}{2}(y^2+2)^{1/2}(2y)$$

$$= y\sqrt{y^2+2}$$

$$A_L = \int_0^1 \sqrt{1+(y^2(y^2+2))}\, dy$$

$$= \int_0^1 \sqrt{y^4+2y^2+1}\, dy = \int_0^1 \sqrt{(y^2+1)^2}\, dy$$

$$= \int_0^1 y^2+1\, dy = \left(\frac{y^3}{3}+y\right)\Big|_0^1$$

$$= \left(\frac{1}{3}+1\right) - \left(\frac{0}{3}+0\right) = \frac{4}{3}$$

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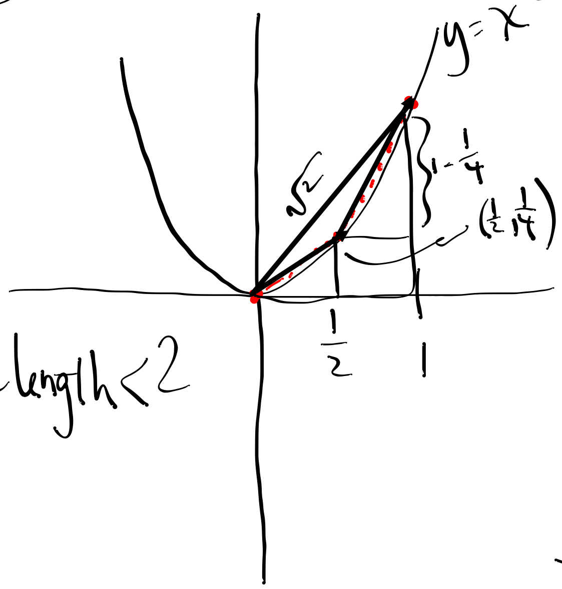
7.4) Arc Length



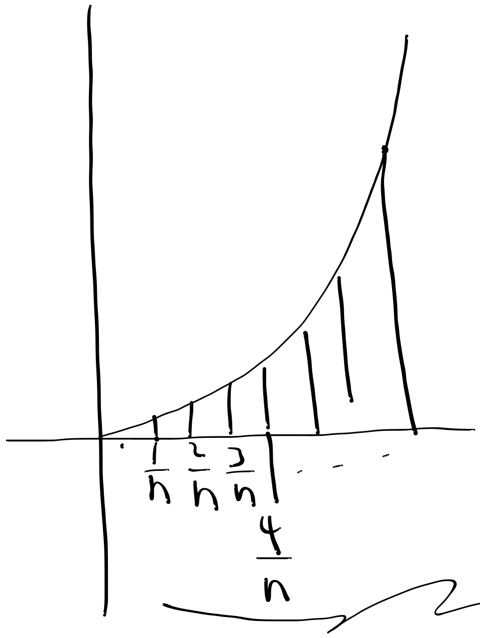
$\sqrt{2} <$ arclength < 2

Arclength $\approx$

$$\sqrt{\left(\frac{1}{2}\right)^2+\left(\frac{1}{4}\right)^2}+\sqrt{\left(\frac{1}{2}\right)+\left(1-\frac{1}{4}\right)^2}$$

$$=\sqrt{\frac{5}{16}}+\sqrt{\frac{13}{16}}$$

$$\approx 1.460$$



Arc Length $\approx$

$$\sum_{k=1}^{n} \sqrt{(\Delta x_k)^2 + \underbrace{(f(x_k) - f(x_{k-1}))^2}_{(\Delta y_k)^2}}$$

shazam-arama #1

If we assume f(x) to f^n's that are continuous... and differentiable; ...

the MVT assures us that there is an x_k^*

with $\frac{f(x_k) - f(x_{k-1})}{\Delta x_k} = f'(x_k^*)$

And so... $f(x_k) - f(x_{k-1}) = f'(x_k^*) \cdot \Delta x_k$

So Arc length is (approx)

$$\sum_{k=1}^{n} \sqrt{(\Delta x_k)^2 + \left(f'(x_k^*)\cdot\Delta x_k\right)^2}$$

$$= \sum_{k=1}^{n} \sqrt{(\Delta x_k)^2}\sqrt{1+\left[f'(x_k^*)\right]^2}$$

$$= \sum_{k=1}^{n} \sqrt{1+\left[f'(x_k^*)\right]^2}\,\Delta x_k$$

$$\text{Arc Length} = \lim_{n\to\infty}\sum_{k=1}^{n} \underbrace{\sqrt{1+f'(x_k^*)^2}}_{\text{thing 1} \to \text{Not } 0}\;\underbrace{\Delta x_k}_{\text{thing 2} \to 0}$$

$$\text{Arc Length} = \int_a^b \sqrt{1+\left[f'(x)\right]^2}\,dx$$

$$\text{ArcLength} = \int_a^b \sqrt{1+[f'(x)]^2}\,dx$$

Find arc length 1) along $y=3$ from $x=1$ to $x=4$.

2) along $y=2x-3$ from $x=1$ to $x=4$.

3) along $y=x^2$ from $x=2$ to 3

$$\text{Arc Length} = \int_a^b \sqrt{1+[f'(x)]^2}\,dx$$

Find arc length 1) along $y=3$ from $x=1$ to $x=4$.

$y=3$

$y'=0$

$$A_L = \int_1^4 \sqrt{1+0^2}\,dx = \int_1^4 1\,dx$$
$$= x\Big|_1^4 = 4-1 = 3$$

2) along $y=2x-3$ from $x=1$ to $x=4$.

$y=2x-3$

$y'=2$

$$AL = \int_1^4 \sqrt{1+2^2}\,dx = \int_1^4 \sqrt{5}\,dx$$
$$= \sqrt{5}\,x\Big|_1^4 = 4\sqrt{5} - 1\sqrt{5} = 3\sqrt{5}$$

3) along $y=x^2$ from $x=2$ to 3

$y=x^2$

$y'=2x$

$$AL = \int_2^3 \sqrt{1+(2x)^2}\,dx$$
$$= \int_2^3 \sqrt{1+4x^2}\,dx = \text{fnInt}((1+4x^2)^{\frac{1}{2}}, X, 2, 3)$$

$$\approx 5.100$$

Arc Length

of $y=x^2$ from 0 to 1

$$= \int_0^1 \sqrt{1+4x^2}\, dx \approx 1.478$$

to find $★ = \int \sqrt{1+4x^2}\,dx$

Extra special college stuff

Let $x = \frac{1}{2}\tan\theta$

$dx = \frac{1}{2}\sec^2\theta\, d\theta$

$$★ = \int \sqrt{1+4\left(\frac{\tan^2\theta}{4}\right)}\left(\frac{1}{2}\sec^2\theta\, d\theta\right)$$

$$\frac{1}{2}\int \sqrt{1+\tan^2\theta}\ \sec^2\theta\, d\theta = \frac{1}{2}\int \sec\theta \cdot \sec^2\theta\, d\theta$$

$$= \frac{1}{2}\int \frac{1}{\cos^3\theta}\, d\theta = \frac{1}{2}\int \frac{\cos\theta\, d\theta}{\cos^4\theta}$$

$\sin^2+\cos^2=1$

$\tan^2+1=\sec^2$

$$= \frac{1}{2}\int \frac{\cos\theta}{(1-\sin^2\theta)^2}\, dt = \frac{1}{2}\int \frac{1}{(1-u^2)^2}\, du$$

$u = \sin\theta$

$du = \cos\theta\, d\theta$

$$= \frac{1}{2}\int \frac{1}{(1-u)^2(1+u)^2}\, du$$

$$= \frac{1}{2}\int \frac{A}{1-u} + \frac{B}{(1-u)^2} + \frac{C}{(1+u)} + \frac{D}{(1+u)^2}\, du$$

$$A(1-u)^1(1+u)^2 + B(1+u)^2 + C(1-u)^2(1+u) + D(1-u)^2 = 1$$

$u=1$ $\quad 4B = 1 \Rightarrow B = \frac{1}{4}$

$u=-1$ $\quad 4D = 1 \Rightarrow D = \frac{1}{4}$

$$A(1-u)(1+u)^2 + \frac{1}{4}(1+u)^2 + C(1-u)^2(1+u) + \frac{1}{4}(1-u)^2 = 1$$

u^2+2u+1
$1-u$
$-u^3-2u^2-u$
$+u^2+2u+1$
$-u^3-u^2+u+1$

$$(-A+C)u^3 + \left(-A-C+\frac{1}{2}\right)u^2 + (A-C)u + \left(\frac{1}{2}+A+C\right) =$$

$-A+C=0$

$-A-C=-\frac{1}{2}$ $\quad A = -\frac{1}{4} \quad C = -\frac{1}{4}$

$-2A = -\frac{1}{2}$

$$= \frac{1}{2}\int -\frac{1}{4}\left(\frac{1}{1-u} + \frac{1}{1+u}\right) + \frac{1}{4}\left(\frac{1}{(1-u)^2} + \frac{1}{1+u^2}\right) du$$

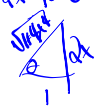
$$= \frac{1}{8}\left(\ln|u-1| + \ln|u+1| + \frac{+1}{1-u} - \frac{1}{1+u}\right) + C$$

$$= \frac{1}{8}\left(\ln|(\sin\theta-1)(\sin\theta+1)| + \frac{1}{1-\sin\theta} - \frac{1}{1+\sin\theta}\right) + C$$

$$= \frac{1}{8}\ln|\cos^2\theta| + \frac{2\sin\theta}{8\cos^2\theta} + C$$

$x = \frac{1}{2}\tan\theta$

$2x = \tan\theta$



$$= \frac{1}{8}\ln\frac{1}{1+4x^2} + \frac{\frac{2x}{\sqrt{1+4x^2}}}{4\,\frac{1}{\sqrt{1+4x^2}}} + C =$$

7.4/3) $y = 3x^{3/2} - 1 \quad [0,1]$

$$y' = \frac{9}{2}x^{1/2}$$

$$AL = \int_0^1 \sqrt{1 + \frac{81}{4}x}\,dx = \left(\frac{4}{81}\right)\left(\frac{2}{3}\left(1 + \frac{81x}{4}\right)^{3/2}\right)\Bigg|_0^1$$

$$= \frac{8}{243}\left(\left(\frac{85}{4}\right)^{3/2} - 1\right)$$

Feb 15-12:51 PM